Birkbeck
(University of London)

MSc Examination

Department of Computer Science and Information Systems

KNOWLEDGE REPRESENTATION AND REASONING (COIY027H7)

CREDIT VALUE: 15 credits

Date of examination: Wednesday, 6th June 2012
Duration of paper: 14:30–16:30

There are FIVE questions in this paper, each worth 25 marks.
Candidates should answer FOUR questions.
Calculators are not permitted.

1. Consider the following argument. "If I did the homework, or I am very clever, then I pass the exam. I do not pass the exam unless I did the exercise. I pass the exam, but I did not do the homework. Therefore I am very clever and I did the exercise."

   (a) Formalise the above argument. (10 marks)

   (b) Determine whether the argument is valid. (15 marks)

2. A frame $\mathcal{F} = (W, R)$ is called *Euclidean* if

$$\forall s \forall t \forall u((sRt \wedge sRu) \rightarrow tRu).$$

   Consider the following formula

$$\text{EUCLID:} \qquad \Diamond\varphi \rightarrow \Box\Diamond\varphi.$$

   (a) Prove that the formula EUCLID is valid in Euclidean frames. (15 marks)

   (b) Show that the formula EUCLID is not valid in general. (10 marks)

3. Consider the following variant of the muddy children puzzle. The father's (true) public announcement is:

   "The number of muddy children and the number of clean children are different."

   In each round the father asks:

   "Raise your hand if you know that you are muddy or you know that you are clean."

   Assume that all children are healthy, intelligent and truthful.

   (a) Assume that there are four children. Describe a model representing the children's knowledge after the father's public announcement. (10 marks)

   (b) Determine what happens in the first and second rounds of the game when there are four children. (Note that your answer should depend on the number of muddy children.) (15 marks)

4. The semantics of the *difference operator* D is given as follows. Let $\mathcal{M} = (S, R, V)$ be a model with possible worlds S, accessibility relation $R \subseteq S \times S$ and valuation V. For $s \in S$ and formula φ,

$$\mathcal{M}, s \models \mathsf{D}\varphi$$

if and only if

$$\mathcal{M}, t \models \varphi \text{ for some } t \in S \text{ such that } s \neq t \text{ and } sRt$$

(a) Let

$$S = \{s_0, s_1, s_2\}$$
$$R = \{(s_0, s_0), (s_0, s_1), (s_0, s_2), (s_1, s_0), (s_1, s_1), (s_2, s_1)\}$$
$$V(p) = \{s_0, s_2\} \text{ and } V(q) = \{s_1, s_2\}$$

Determine at which worlds in S the formula $\mathsf{D}(p \wedge \mathsf{D}q)$ is true in the model (S, R, V). (10 marks)

(b) Give a syntactic definition of $\Diamond\varphi$ in reflexive models using D, that is, find a formula ψ using only D and propositional connectives such that, for every reflexive model $\mathcal{M}$ and world s,

$$\mathcal{M}, s \models \Diamond\varphi \text{ if and only if } \mathcal{M}, s \models \psi$$

Justify your answer. (10 marks)

(c) Give a syntactic definition of $\mathsf{D}\varphi$ in irreflexive models (where sRs does not hold for any world s) using $\Diamond$, that is, find a formula ψ using only $\Diamond$ and propositional connectives such that, for every irreflexive model $\mathcal{M}$ and world s,

$$\mathcal{M}, s \models \mathsf{D}\varphi \text{ if and only if } \mathcal{M}, s \models \psi$$

Justify your answer. (5 marks)

5. Consider the following TBox Σ:

$$\text{Mother} \equiv \exists\text{hasChild}.\top \sqcap \text{Female}$$
$$\text{Father} \equiv \exists\text{hasChild}.\top \sqcap \text{Male}$$
$$\text{Grandmother} \equiv \exists\text{hasChild}.(\text{Mother} \sqcup \text{Father}) \sqcap \text{Female}$$
$$\text{Grandfather} \equiv \exists\text{hasChild}.(\text{Mother} \sqcup \text{Father}) \sqcap \text{Male}$$

We define:

$$\text{Grandparent}_1 \equiv \exists\text{hasChild}.(\exists\text{hasChild}.\top)$$
$$\text{Grandparent}_2 \equiv \text{Grandmother} \sqcup \text{Grandfather}$$

(a) Determine whether the following subsumptions are true:

$$\Sigma \models \text{Grandparent}_1 \sqsubseteq \text{Grandparent}_2$$
$$\Sigma \models \text{Grandparent}_2 \sqsubseteq \text{Grandparent}_1$$

Justify your answer. (15 marks)

(b) Add a formula to Σ so that the two definitions of Grandparent become equivalent. Explain your answer. (10 marks)

